Methods

1 Preparation of Ovalbumin-Alginate Capsules

This is based on the paper by Levy and Edwards-Levy 1996 [1].

The key to getting good capsules with thin membranes is to always use fresh $CaCl_2$ as the sphericity of the capsules rapidly decreases when it is reused.

1. Prepare the initial aqueous solution by dissolving sodium alginate (1 %), a polysaccharidic ester and a protein in distilled water.

   Polysaccharidic ester: 2% propylene glycol alginate (PGA) or 3% pectin

   Protein: 5% human serum albumin (HSA), 8% ovalbumin, 10% hemoglobin or 5% Prosobel

2. Add the resulting solution drop wise to a 10% $CaCl_2$ solution in a ration of 1 to 5 under continued stirring.

3. Rinse beads several times in water.

4. Add NaOH to the suspension of the beads and stir for 15 minutes. Neutralize with HCL and stir for a further 15 minutes.

5. Rinse beads several times in water.

6. Place the coated beads in a 10% solution of sodium citrate solution. Stir for 10 minutes.

7. Rinse beads several times in water.

2 Compression Test

2.1 Feng and Yang Equations

For the undeformed configuration, the spherical coordinates (r, θ, ψ) are used and for the deformed configuration the cylindrical coordinates (ρ, θ, η). The governing equations are separately applied to the contact and the non-contact region (the governing equations are written in the from found in Lardner and Pujara 1980).

Contact Region

$$\lambda_1^{'} = -\frac{\lambda_1}{\lambda_2 sin\psi}\left(\frac{f_3}{f_1}\right) - \left(\frac{\lambda_1 - \lambda_2 cos\psi}{sin\psi}\right)\left(\frac{f_2}{f_1}\right) \tag{1}$$

$$\lambda_2^{'} = -\frac{\lambda_1 - \lambda_2 cos\psi}{sin\psi} \tag{2}$$

Non-contact Region

$$\lambda_1^{'} = \left(\frac{\delta cos\psi - \omega sin\psi}{sin^2\psi}\right)\left(\frac{f_2}{f_1}\right) - \left(\frac{\omega}{\delta}\right)\left(\frac{f_3}{f_1}\right) \tag{3}$$

$$\delta^{'} = \omega \tag{4}$$

$$\omega^{'} = \frac{\lambda_1^{'}\omega}{\lambda_1} + \frac{\lambda_1^2 - \omega^2}{\delta}\frac{T_2}{T_1} - \frac{\lambda_1\left(\lambda_1^2 - \omega^2\right)^{1/2} Pr_0}{T_1} \tag{5}$$

where the prime indicates differentiation with respect to ψ, δ is defined by $\delta = \lambda_2\ sin\psi$ and the functions are defined as

$$f_1 = \frac{\partial T_1}{\partial \lambda_1} \quad ; \quad f_2 = \frac{\partial T_1}{\partial \lambda_2} \quad ; \quad f_3 = T_1 - T_2 \tag{6}$$

The boundary conditions for this problem are

$$\psi = \Gamma\ :\quad \eta^{'} = 0 \text{ or } \delta^{'} = \lambda_1 \quad \psi = \Gamma\ :\quad (\lambda_1)_c = (\lambda_1)_{n-c} \quad \psi = \Gamma\ :\quad (\lambda_2)_c = (\lambda_2)_{n-c} \tag{7}$$

$$\psi = 0\ :\quad \lambda_1 = \lambda_2 = \lambda_0 \qquad \psi = \frac{\pi}{2}\ :\quad \delta^{'} = 0 \tag{8}$$

where Γ is the contact angle. The volume of the inflated membrane after contact can be found

$$V = 2\pi r_0^3 \int_{\Gamma}^{\frac{\pi}{2}} \left(\lambda_1^2 - \delta'^2\right)^{1/2} \delta^2\, d\psi \tag{9}$$

The pressure P inside the membrane after contact then is

$$P = \frac{2P_0\lambda_s^3}{3\int_{\Gamma}^{\frac{\pi}{2}}\left(\lambda_1^2 - \delta'^2\right)^{1/2}\delta^2\, d\psi} \tag{10}$$

2.2 Feng and Yang procedure

The numerical computation proceeds as follows.

1. Assign a value to λ_s. The pressure p_0 can be calculated from an equation involving the membrane constitutive model.

2. Assume a value for λ_0. The boundary condition 8 provide initial conditions for equations 1 - 2.

3. Apply an integration method to equations 1 - 2 simultaneously to compute successive points in the contact region until psi equals the preassigned contact angle Γ.
4. The boundary conditions 7 provide initial conditions for the governing differential equations 3 - 5 for the non-contact region. However, when the integration in the free region is begun, we made $\delta^{'}$ a little less than λ_1, therby creating a disturbance.
5. Guess a value for p
6. Apple a numerical integration scheme to equations 3 - 5 simultaneousness to compute successive points in the non-contact region until $\delta^{'}$ equals zero. Check the boundary condition $\psi = \frac{\pi}{2}$: $\delta^{'} = 0$. If this is not satisfied, reassume a value for p until it is satisfied.
7. Check equation 10. If it is not satisfied, reassume a value for λ_0 and repeat steps 2-7 until it is satisfied.

2.3 Constitutive Equation

To determine the functions f_1 - f_3 a constitutive equation for the material properties is needed.

2.4 Mooney Model

This models an isotropic incompressible materials with material constants C_1 and C_2 with the same dimensions as stress. The strain invariant $\alpha = C_2/C_1$. The tensions T_1 and T_2 are then given by

$$T_1 = 2hC_1 \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{\lambda_1^3 \lambda_2^3} \right) \left(1 + \alpha\lambda_2^2\right) \;\; ; \quad T_2 = 2hC_1 \left(\frac{\lambda_2}{\lambda_1} - \frac{1}{\lambda_1^3 \lambda_2^3} \right) \left(1 + \alpha\lambda_1^2\right) \tag{11}$$

and hence

$$f_1 = \frac{\partial T_1}{\partial \lambda_1} = 2hC_1 \left(1 + \alpha\lambda_2^2\right) \left(\frac{1}{\lambda_2} + \frac{3}{\lambda_1^4 \lambda_2^2} \right) \tag{12}$$

$$f_2 = \frac{\partial T_1}{\partial \lambda_2} = 2hC_1 \left[\left(1 + \alpha\lambda_2^2\right) \left(-\frac{\lambda_1}{\lambda_2^2} + \frac{3}{\lambda_1^3 \lambda_2^4} \right) + \left(\frac{\lambda_1}{\lambda_2} - \frac{1}{\lambda_1^3 \lambda_2^3} \right) 2\alpha\lambda_2 \right] \tag{13}$$

$$f_3 = T_1 - T_2 = 2hC_1 \left[\frac{\lambda_2}{\lambda_1} - \frac{\lambda_1}{\lambda_2} + \alpha \left(\frac{1}{\lambda_1^3 \lambda_2} - \frac{1}{\lambda_1 \lambda_2^3} \right) \right] \tag{14}$$

2.4.1 Neo-Hookian Model

Assumes membrane is a sheet of incompressible rubber-like material.

$$T_1 = \frac{G_s}{\lambda_1 \lambda_2} \left(\lambda_1^2 - \frac{1}{(\lambda_1 \lambda_2)^2} \right) \;\; ; \quad T_2 = \frac{G_s}{\lambda_2 \lambda_1} \left(\lambda_2^2 - \frac{1}{(\lambda_2 \lambda_1)^2} \right) \tag{15}$$

and hence

$$f_1 = \frac{\partial T_1}{\partial \lambda_1} = -\frac{G_s}{\lambda_1^2 \lambda_2} \left(\lambda_1^2 - \frac{1}{(\lambda_1 \lambda_2)^2} \right) + \frac{G_s}{\lambda_1 \lambda_2} \left(2\lambda_1 + 2\frac{1}{\lambda_1^3 \lambda_2^2} \right) \tag{16}$$

$$f_2 = \frac{\partial T_1}{\partial \lambda_2} = -\frac{G_s}{\lambda_2^2 \lambda_1} \left(\lambda_1^2 - \frac{1}{(\lambda_1 \lambda_2)^2} \right) + \frac{G_s}{\lambda_1 \lambda_2} \left(2\frac{1}{\lambda_2^3 \lambda_1^2} \right) \tag{17}$$

$$f_3 = T_1 - T_2 = \frac{G_s}{\lambda_1 \lambda_2} \left(\lambda_1^2 - \lambda_2^2 \right) \tag{18}$$

2.4.2 Skalak law

$$T_1 = G_s \left(\frac{\lambda_1}{\lambda_2} (\lambda_1^2 - 1) + C \lambda_1 \lambda_2 ((\lambda_1 \lambda_2)^2 - 1 \right) \tag{19}$$

2.4.3 Evans and Skalak Law

References

[1] Levy, M.-C. and Edwards-Levy, F. *J. Microencapsulation* **13**(2), 169–183 (1996).